straightforward; however, precise work is required and exceptional care should be exercised in interpretation of results. In Table IV the two photoelastic test results check theory almost exactly; these tests were conducted and interpreted under most exacting conditions. To avoid the masking effects of stress gradients, compensator readings were taken as successive reduction of specimen thickness and the results were extrapolated to the state of stress at the surface of the bore. It is possible that the two photoelastic test results reported in the literature (R = 1.5 and 3.0) were not based on such a procedure, and this may account for their failure to check theory more closely. In particular, the result for the cylinder of R = 1.5 seems to be too low. On the basis of information given in the reference to this work it is estimated, on the basis of stress gradients, that the true K factor for the cylinder is 2.2, which fits into the over-all picture very well.

The only cylinder with side holes that were not circular contained a slot. This slot approximated an ellipse having an axis ratio a/b of 2.0—it exhibited a K factor at the cylinder bore–side hole interface of about 1.7. The theoretical result for a single elliptic side hole is 1.46. Although in this case the theoretical result for a single elliptic side hole cannot properly be compared with the experimental result for a cross-bore hole that is not a true ellipse, the qualitative check is good. Actually the slot should induce more stress concentration effect than an ellipse, as it is intermediate in geometry between the circle and the ellipse.

An examination of the data in Table IV reveals that the predictions are by no means perfect, although they are probably acceptable from an engineering point of view. Work still remains to be done on this problem.

Application of Theory

Examples given below illustrate use of the theory and its application to engineering design.

Example 1. Design of Plain Monobloc Cylinders with Circular Side Holes. If K values are available, it is possible to estimate the elastic-limit pressure for a cylinder containing side holes. Assuming that the cylinder is initially stress-free, the elastic-limit pressure is obtained by substituting maximum principal stress values for a cylinder, corrected for stress concentration, into the von Mises criterion for failure; thus, for a closed-end cylinder (A, Figure 5)

$$\sigma_{h(\max)} = Kp\left(\frac{R^2+1}{R^2-1}\right) \tag{25}$$

$$\sigma_{r(\max)} = -p \tag{26}$$

and

$$\sigma_{z(\max)} = -p \tag{27}$$

These stresses are related by the von Mises criterion to the yield strength of the material in tension, σ_o, as follows:

$$\sigma_o = (\sigma_h^2 + \sigma_r^2 + \sigma_z^2 - \sigma_h\sigma_r - \sigma_r\sigma_z - \sigma_z\sigma_h)^{1/2} \tag{28}$$

Substitution of Equations 25 to 27 in Equation 28 thus gives an expression for the elastic-limit pressure, p_y, of a cylinder,

$$p_y = \sigma_o(R^2 - 1)\ [R^4(K^2 + 2K + 1) + 2R^2(K^2 - 1) + (K^2 - 2K + 1)]^{-1/2} \tag{29}$$

If R = 2 and R_s = 4, from Figure 9, K = 2.47 and by Equation 29

$$p_y = 0.195\ \sigma_o \tag{30}$$

If there were no side hole,

$$p_y = \sigma_o(3)^{-1/2}\left(\frac{R^2-1}{R^2}\right) = 0.432\ \sigma_o \tag{31}$$

For this case the cylinder with the side holes would yield (locally) at a pressure 55% lower than the cylinder without side holes.

If R is increased to 4 in a cylinder with no side holes, then by Equation 31

$$p_y = 0.54\ \sigma_o \tag{32}$$

or an elastic-limit pressure increase of 25% over that obtained for the cylinder with a wall ratio of 2. However, for R = 4 and R_s = 4, K = 2.485, and by Equation 29

$$p_y = 0.258\ \sigma_o \tag{33}$$

or an increase in elastic-limit pressure of about 32%. Thus benefit can be obtained, elastically, in increasing the wall ratio; even though the K values increase with increasing wall ratio, the elastic-limit properties increase at a more rapid rate (due to increase in wall ratio) than the decrease in elastic-limit properties caused by increasing K. For each case, however, the situation should be examined in order that realistic values may be assigned to specific designs.

On the other hand, difficulty could be encountered in a particular case, designed to operate within the elastic range, if additional side holes of different R_s values were used. For example, if R = 2 and R_s = 1, then by Equation 29 and Figure 9

$$p_y = 0.218\ \sigma_o \tag{34}$$

Now if to this cylinder an additional set of side holes is introduced having an R_s value of, say, 10 (small oil hole), then

$$p_y = 0.151\ \sigma_o \tag{35}$$

or a decrease in elastic-limit pressure of 31%.

Example 2. Comparative Design of Cylinders with Circular Side Holes. In this example some comparisons are made relative to the elastic-limit pressures of cylinders of various R values containing circular side holes of various R_s values. The comparison includes a conventional design where the stress concentration factor is assumed to be 6.00.

As in Example 1, Equations 25 to 27 define the state of stress in the cylinders, which when put into Equation 28 gives the elastic-limit pressure defined by Equation 29, where K is determined from Figure 9. The results of some calculations are shown in Table V, where relative elastic limits for cylinders of various geometries are listed. When the side hole ratio, R_s, is infinity it is assumed that the cylinder acts as a plain monobloc with a relative elastic

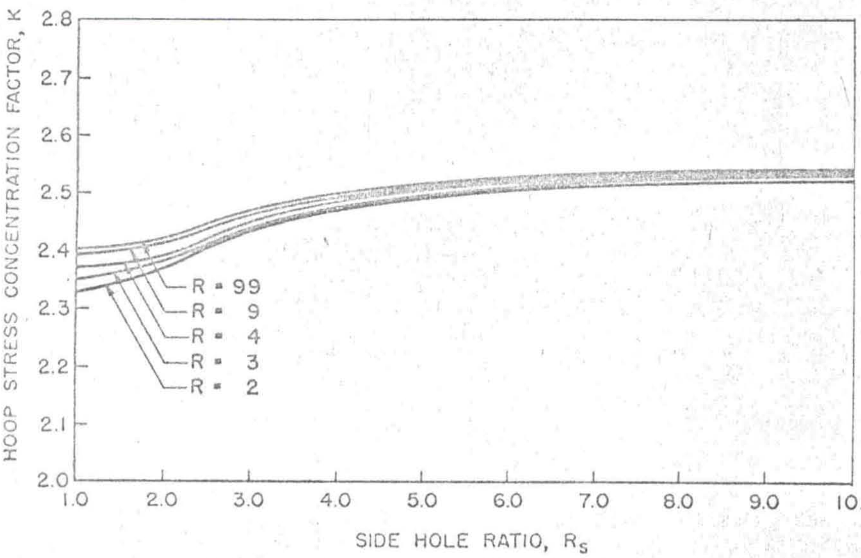

Figure 9. K values for various cylinder geometries